Some game theory models in criminality

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Overview

- Basics of Game Theory
- Crime and punishment.
- A model of game theory for a terrorist situation.
- Other approaches?.

Basics of Game Theory

- Cooperative and Non-cooperative
- Normal form of the game
- Mixed strategies
- Maximin strategies
- Nash Equilibrium
- On strict Nash equilibrium
- Stackelberg equilibrium

Prisoner's dilemma

Two prisoners (the players) during the interrogation have a choice each: whether to betray the other, and thus to decrease the own jail time by, for example, 1 month (as a compensation for the cooperation), while increasing the jail time for the other by, for example, 10 years, or to stay silent. Each of the prisoners is only interested in receiving the least possible sentence. It shall be assumed that the prisoners make their choices (to betray or to stay silent) simultaneously, and they know for sure that their choice cannot affect the choice of the other one.

Prisoner's dilemma

	Prisoner B Stays Silent	Prisoner B Betrays
Prisoner A Stays Silent	Each serves 6 months	Prisoner A: 10 years Prisoner B: goes free
Prisoner A Betrays	Prisoner A: goes free Prisoner B: 10 years	Each serves 5 years

Prisoner's dilemma

	Cooperate	Defect
Cooperate	**3, 3**	**0, 5**
Defect	**5, 0**	**1, 1**

The Postman Always Rings Twice

Tosca

Crime and Punishment

- **Tsebelis, G.** (1990's)
- "Penalty Has No Impact on Crime: A Game Theoretic Analysis" (July 1990). Rationality and Society 2: 255-86
- "Crime and Punishment: Are One-Shot, Two-Person Games Enough?“. (Controversy with W. Bianco and P. Ordeshook June 1990) AmericanPolitical Science Review 84: 569-86
- "Are Sanctions Effective? A Game Theoretic Analysis" (March 1990) Journal of Conflict Resolution 34: 3-28

	Enforce	Do not enforce
Speed	$(a_{11}, \underline{b}_{11})$	$(\underline{a}_{12}, b_{12})$
Do not speed	$(\underline{a}_{21}, b_{21})$	$(a_{22}, \underline{b}_{22})$

The game has a unique Nash equilibrium in mixed strategies (p, q)

- Best response functions:

$$p^*(q) = \begin{cases} 0 & if \;\; q > Q \\ [0,1] & if \;\; q = Q \\ 1 & if \;\; q < Q \end{cases} \qquad Q = {(a_{12} - a_{22})}\Big/{(a_{12} - a_{22} + a_{21} - a_{11})}$$

$$q^*(p) = \begin{cases} 0 & if \;\; p < P \\ [0,1] & if \;\; p = P \\ 1 & if \;\; p > P \end{cases} \qquad P = {(b_{22} - b_{21})}\Big/{(b_{11} - b_{12} + b_{22} - b_{21})}$$

Penalty has no effect on crime!

- The equilibrium level of speeding, P, does not depend on the payoffs to the public, but only on the payoffs to the police.
- Is this model correct?
- Starting point for the proposal of alternative models (sequential, n-person, incomplete information,...)

Table 1. The inspection game

	Inspect	Not inspect
Violate	a_{11}, b_{11}	a_{12}, b_{12}
Not violate	a_{21}, b_{21}	a_{22}, b_{22}

Andreozzi (2004):

Claim 1. In the inspection game in Table 1:

- 1. Increasing penalties (i.e. reducing a_{11}), leaves the frequency P of law violations unchanged and reduces the frequency of inspections Q;
- 2. Increasing incentives for inspectors to play Inspect (i.e. raising b_{11}), leaves the frequency of inspections Q unchanged and reduces the frequency of law infractions P .

Maximin strategies

$$p^+ = \begin{cases} 0 \text{ if } \hat{p}^+ < 0 \\ \hat{p}^+ \text{ if } \hat{p}^+ \in [0,1] \\ 1 \text{ if } \hat{p}^+ > 1 \end{cases} ; \quad q^+ = \begin{cases} 0 \text{ if } \hat{q}^+ < 0 \\ \hat{q}^+ \text{ if } \hat{q}^+ \in [0,1] \\ 1 \text{ if } \hat{q}^+ > 1 \end{cases} .$$

$$(\hat{p}^+, \hat{q}^+) = \left(\frac{a_{22} - a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}}, \frac{b_{22} - b_{12}}{b_{11} - b_{12} - b_{21} + b_{22}}\right)$$

Andreozzi (2004):

Claim 2. If the two players employ their maximin strategies, and the maximin strategies are mixed (that is if p^+, q^+ belongs to *(*0*,* 1*)*) then,

- 1. increasing the severity of punishment will reduce crime;
- 2. increasing b_{11} will not reduce crime but will reduce the frequency with which the inspector plays Inspect.

An alternative model (Cox, 1994)

FIRST: Restrictions on the payoffs to the public:

- 1) The public does not care whether the police enforce or not when it is not speeding:

$$a_{21} = a_{22}$$

- 2) The public garners a specific benefit, *s*, from speeding (adrenaline, power feeling,…)
- 3) The public incurs a specific fine, *f*, when caught speeding

Normalization:

$$a_{21} = 0, \quad a_{11} = s - f, \quad a_{12} = s, \quad a_{21} = a_{22} = 0$$

The public likes to speed (s>0) but not if it knows it will be caught (s<f); if the public chooses not to speed, it does not care whether the police enforce or not

SECOND:

- n motorists (at least 1)
- s_i will represent the benefit that the ith motorist garners from speeding

THIRD

- Distribution of *s, G(x)* equals the proportion of motorists who derive a benefit from speeding less than or equal to *x*.

FOURTH

- Police move first, anticipating the (optimal) response of the public.

- Optimal response of the ith motorist:

$$p_i(q) = \begin{cases} 0 & if \;\; qf > s_i \\ [0,1] & if \;\; qf = s_i \\ 1 & if \;\; qf < s_i \end{cases}$$

The police determines q taking into account that the expected proportion of the population speeding, given q, is *1-G(qf)*.

- Assuming *G* uniform in *[0,F],* where *F* can be interpreted as the largest expected fine that any motorist would be to pay for the pleasure of speeding.
- Computes the Stakelberg equilibrium (*p*, q*)*
- The partial derivative of *p** with respect *f* equals *–A/F,* where

$$A = (b_{22} - b_{12})/2(b_{11} - b_{12} + b_{22} - b_{21})$$

- Thus the equilibrium rate of speeding is unaffected by the size of the fine if and only if A=0
- If the police prefer that motorists not speed rather than speed, when they do not enforce the law, then the equilibrium level of speeding will decline with increases in the fine.
- More general sufficient conditions.

Andreozzi, L. (2004). Rewarding policemen increases crime. Another surprising result from the inspection game.

- The inspector (the police) acts as a Stakelberg leader.
- Increasing, inspector’s incentives to enforce the law increase the frequency of law infractions.

Rimawan, P. (2007). Does Punishment Matter? A Refinement of the Inspection Game.

- The severity of punishment may affect the offending behaviour of individuals.
- The impact of increasing the severity of punishment on reducing individuals’ offending behaviour is less certain than that of instigating crime prevention programs.

Some conclusions

- Game theory is relevant in analysing crime deterrence.
- Controversial results.
- Not universal models
- Lack of empirical evidence
- Too much work to do